The Pendulum

Background

The pendulum is a body suspended from a fixed point so as to swing freely to and fro under the action of gravity. Its regular motion has served as the basis for measurement, as recognized by Galileo. Huygens applied the principle to clock mechanisms. Other applications include seismic instrumentation and the use by NASA to measure the physical properties of space flight payloads. The underlying equation is at the heart of many problems in structural dynamics. Structural dynamics deals with the prediction of a structure's vibratory motions. Examples include the smoothness or bounciness of the car you ride in, the motion that you can see if you look out of the window of an airplane in a bumpy flight, the breaking up of roads and buildings in an earthquake, and anything else that crashes, bounces or vibrates. With this pendulum motion as point of departure, complex structures can be analyzed.

The pendulum serves as an illustration of Newton's Second Law, which states that for every force there is an equal and opposite reaction. The simpler experiments illustrate another of Newton's laws, namely, that a body in motion continues in motion unless acted upon by another force. The pendulum offers an extensive array of experiments that can be done using easy to obtain, inexpensive materials. The measurements require no special skills and equipment. The graphical results of each experiment are given, and can be compared to the results calculated from a simple equation if desired.

The pendulum is depicted as:

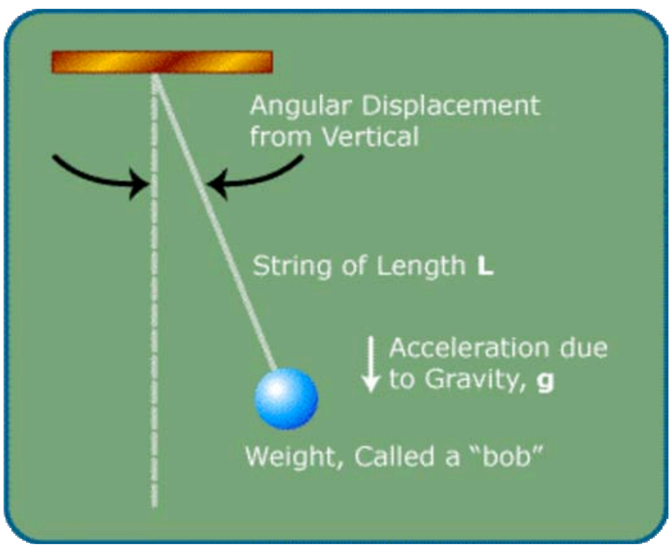

Figure 1

Figure 2

Pendulum, period and independence of amplitude

The pendulum will swing through small displacements from its rest position at a fixed period (i.e., move from one side and return to that side) that is dependent on the length of the string and the acceleration of gravity. This is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Notice that the weight of the bob does not show up in this equation. This means that no matter what the weight, a bob that is suspended on a certain length of string will take the same time to complete a prescribed motion, for example, 10 cycles. Notice that the angle that the pendulum swings through does not show up in the equation either! This means that the period is independent of the angle of swing. This is true whether you swing the pendulum a little bit or a larger amount, say 15 degrees (for very large angles, you will observe a difference).

The following is description of experiments that will illustrate these points. We will describe what you can measure, how you can present the results, and what you can calculate to verify the theory.

Equipment needed:

1. Strings of several lengths, ranging from 20 inches up to 60 inches.
2. One or more lead sinker weights of about an inch in dimension.
3. A stopwatch for timing the swinging of the weights.
   (A video camcorder that shows time in the picture it recorded is ideal).
4. A container (tub size) for water and oil
5. A means of securing the top of the string so that the pendulum can be hung securely.

A lead sinker makes an ideal bob for the pendulum. A plumb bob can also be used. The sinker can be inexpensively purchased in a sporting goods store and the bob in a hardware store. Any solid will do as long as it is heavy compared to the string on which it is suspended.

Experiment 1

Attach weight to the shortest string, then attach the other end of the string to a support, such as shown in Figure 2. Measure the length of the string. Lift the bob (keeping the string taut) so that the string angle from the vertical is about 15 degrees (about as much as you see in the diagram of the pendulum in Figure 1). Let go of the bob without pushing it. Note the time when it is released, and count the bob's return for 10 cycles. Note the time when the 10th cycle is completed. The period is this time is divided by ten. (You can substitute another number for 10. Doing it for only one cycle is not recommended due to the fact that it introduces large measurements errors in the recording of time). Repeat the measurement several times and take the average result. Then repeat the experiment for several lengths ranging from 20-60 inches. If you do this and plot your data on a graph of period (time) as a function of pendulum string length, your data should fall on a curve as shown in Figure 3. Finally, calculate the periods from equation 1:

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(Equation 1)}$$

and compare the results to your experimental graph. You should also try different weights on the strings to see if the period changes (for a given length). See Figure 2, where two different weights are shown on strings of the same length.

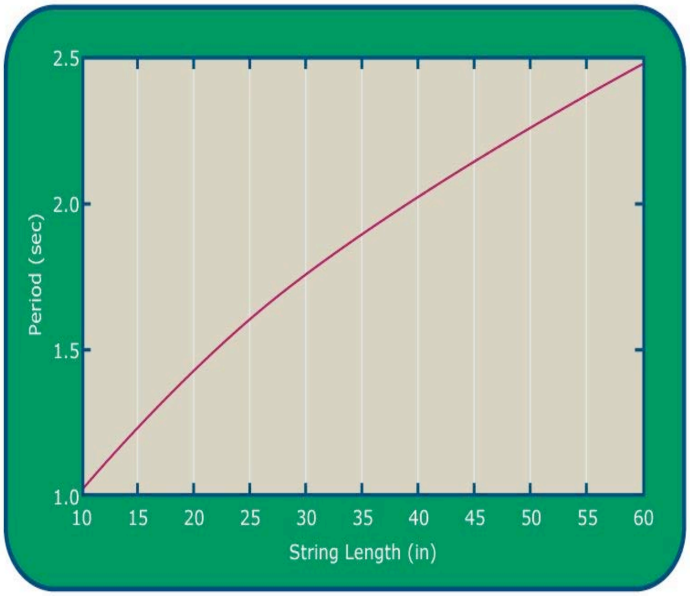

Figure 3

Experiment 2
Have a friend swing on a swing set through a small amplitude (again say 15 degrees). Measure the time for 10 complete swings. Divide the time that you measured by 10 to get the period in seconds. Now, by reading values off the following graph in Figure 4, you can determine length of the swing. The weight of the person helping you by swinging did not matter!

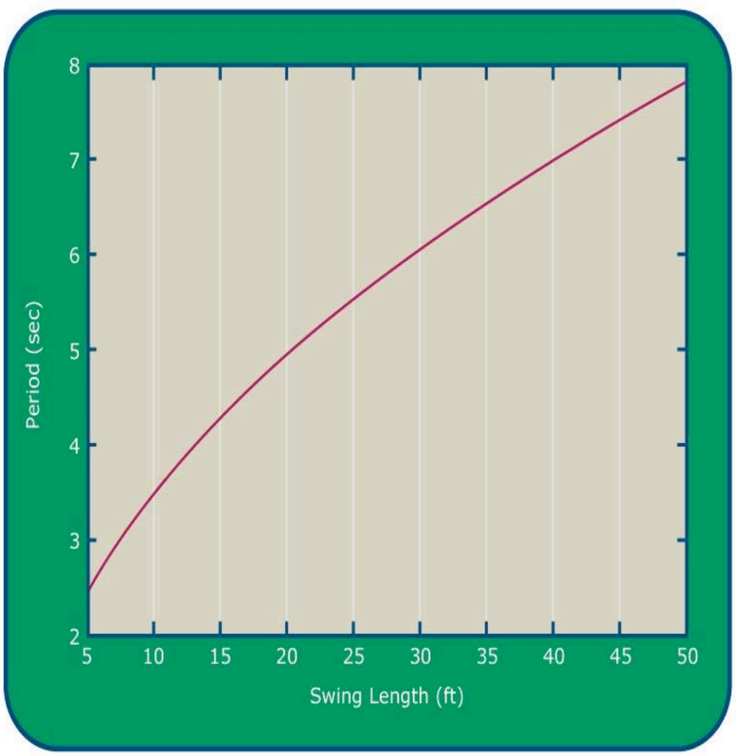

Figure 4

To use this graph, enter the time from the left, and read the length from below.

Experiment 3
A final elementary test we can try is to swing the pendulum through different amplitudes and measure the period. The results of this experiment will show that the period does not change as we change the amplitude of oscillation, for small angles. Graphing these results would look like Figure 5.

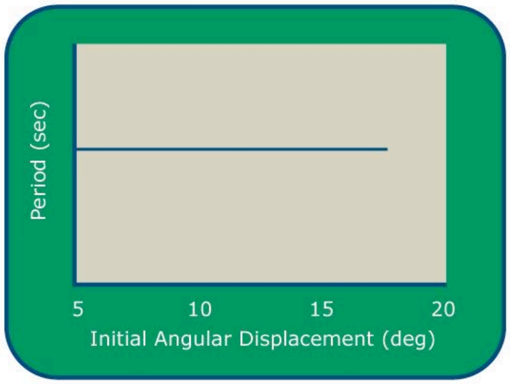

Figure 5

Note: If the release angle is equal to and greater than 45 degrees, the period will change significantly.

For the set of elementary experiments, you may find that the results do not fall exactly on the lines of the two graphs, or that the period is not exactly independent of the pendulum amplitude. The reasons for this are various. In any experiment there will be experimental error. Maybe you did not record time accurately or measure the string accurately. Your care in doing the measurements will help to reduce errors. Another possibility is that you are violating some assumptions that were used in developing the underlying theory. One example of this is that the angle must be small, and you may have swung the pendulum through too great an angle. What this shows is that when we perform an experiment to verify a theoretical calculation, we are actually looking at a simplified view (an approximation) of the real world both in the experiment and in the theory.

Effects of Friction On Pendulum Behavior
If you did the first experiment, you noticed that the pendulum gradually slows down and eventually stops. This is no surprise. The cessation of motion is the result of friction, which did not influence what we were illustrating in the previous experiments. The effect of friction did not show up in equation 1. We will now explore some of the effects of friction. The amplitude of the pendulum swing decreases with each cycle (i.e., with increasing time) because of friction, according to the amount of damping that is present. If there is little damping, as there is for a pendulum swinging in air, the motion will gradually decrease in amplitude as shown in Figure 6. The pendulum will swing for a long time.

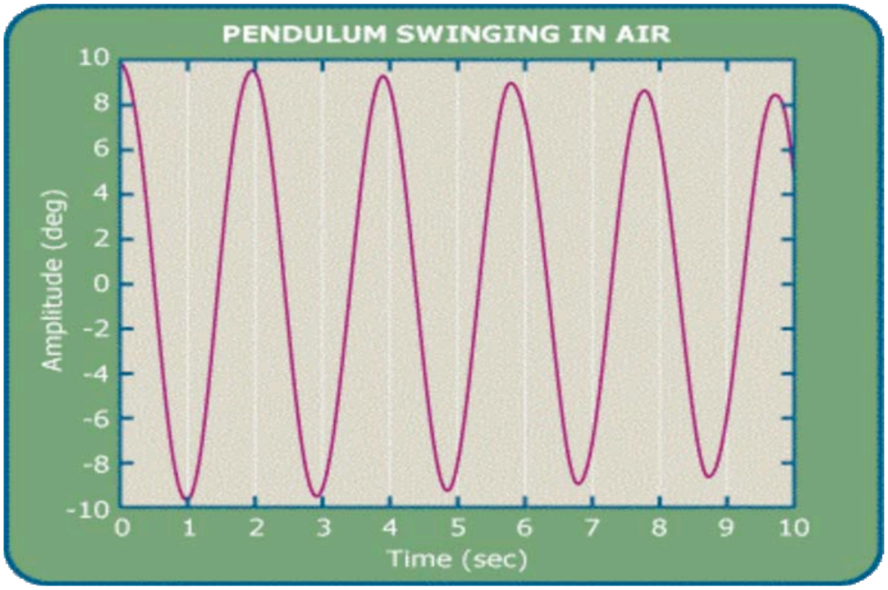
PENDULUM SWINGING IN AIR
Amplitude (deg)
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
1
2
3
4
5
6
7
8
9
10
Time (sec)

Figure 6

Advanced Experiments

Experiment 4: Friction effects for pendulum swinging in air
Choose one of the strings from Experiment 1 and attach the weight to it. Pull the bob to about 15 degrees from the vertical and release it. Count the number of cycles and the time required for the pendulum to come to a complete stop, and record the data. This will take a while! Calculate the period as the time divided by the number of cycles.

Experiment 5: Friction effects for pendulum in water
Use the same pendulum from Experiment 4 and place a container of water, as shown in Figure 7, such that the pendulum will swing in the water. Pull the bob back to about 15 degrees from the vertical and release it. Again, count the number of cycles and time required for the pendulum to come to a complete stop, and record the data. Compare to results of experiment 4. Why does water exert a greater frictional force than air on the pendulum? Is the period different compared to experiment 4?

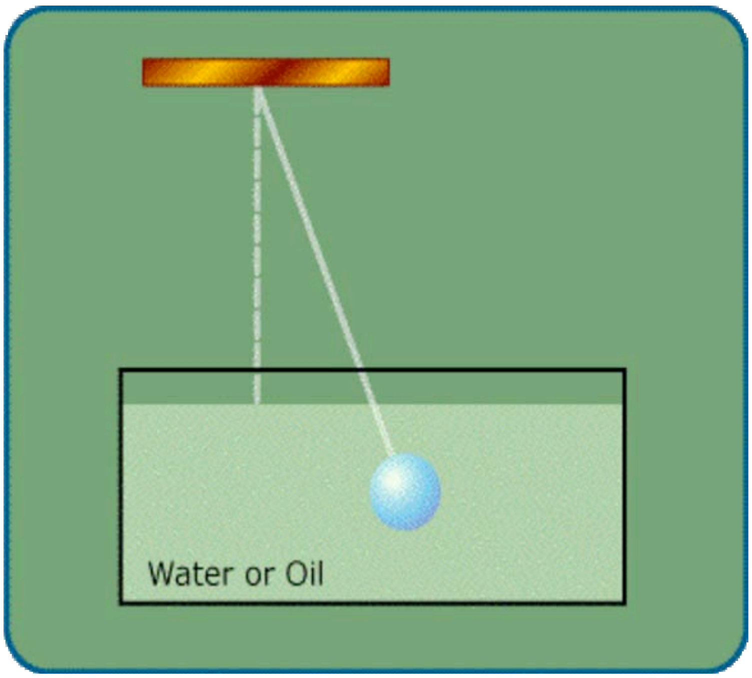

Figure 7

Experiment 6: Friction effects for the pendulum in oil
Repeat Experiment 5, but instead of water fill the container with motor oil. Again, pull the pendulum back 15 degrees, release it, and count the number of cycles and the time required for the oscillations to completely stop. Calculate the period as time divided by the number of cycles. Construct a table comparing results for Experiments 4, 5, and 6. The table should list the number of cycles and time required for the pendulum to completely stop, and the period for air, water, and motor oil. Which liquid exerts the greater frictional force on the pendulum, and why? Does friction effect the period?