NATURAL FREQUENCIES OF COMPOSITE BEAMS

By Tom Irvine
Email: tomirvine@aol.com

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Beam Simply-Supported at Both Ends

The method in this report is based on Reference 1.

Consider a simply-supported beam with length L, as shown in Figure 1. The cross-section consists of two materials, as shown in Figure 2.

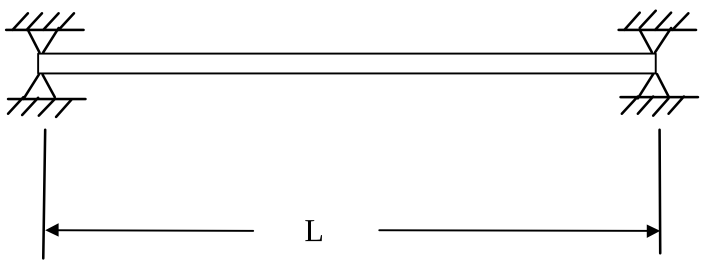

Figure 1.

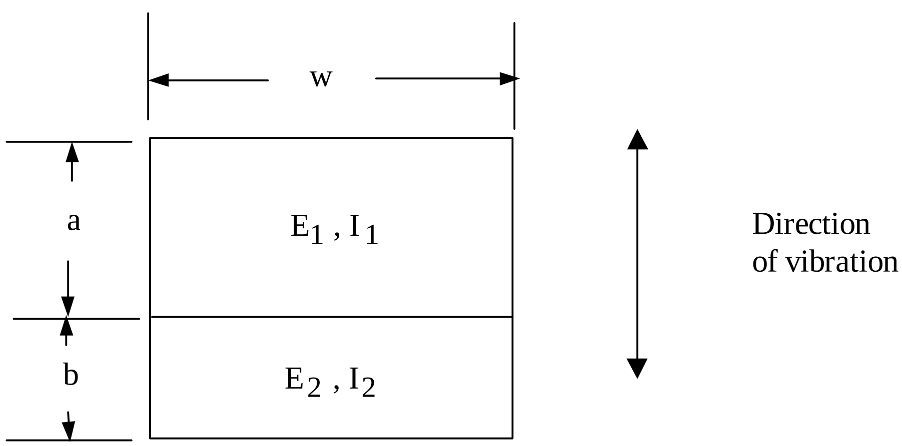

Figure 2.

The fundamental bending frequency for a beam simply-supported at both ends is

$$f_n = \frac{\pi}{2L^2}\sqrt{\frac{EI}{\rho}} \qquad (1)$$

where

E is the modulus of elasticity.
I is the area moment of inertia.
ρ is the mass density, mass per length.

Example

Consider a simply-supported laminated beam, with the cross-section shown in Figure 3.

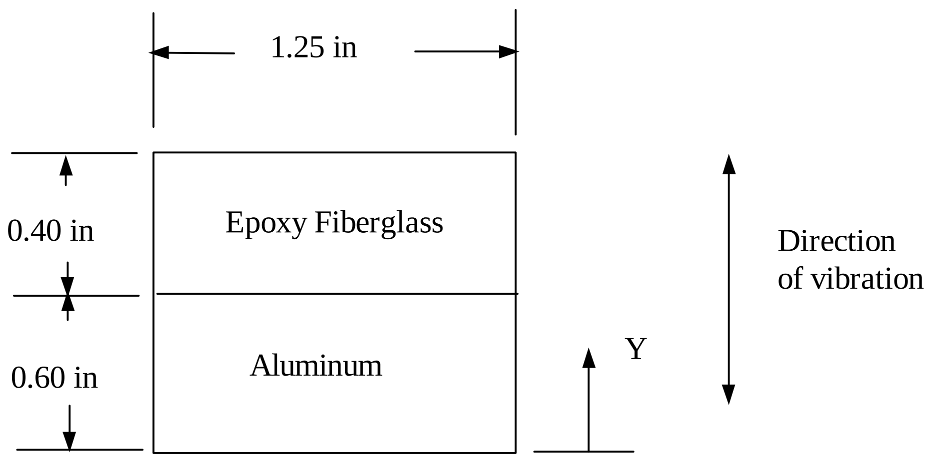

Figure 3.

The material properties are given in Table 1.

Table 1. Material Properties		
Material	E $[lbf/in^2]$	m $[lbm/in^3]$
Epoxy Fiberglass	2.0 (10^6)	0.065
Aluminum	10.0 (10^6)	0.100

The center of mass of each material is referenced to the bottom side of the lower section. Let Y be the center of mass for each section.

Y_1 = 0.80 inch, for epoxy section

Y_2 = 0.30 inch, for aluminum section

Now calculate an overall centroid $\overline{Y}$, weighted in terms of the product of area and elastic modulus.

Table 2. AE Centroid					
Section	Area $\left(in^2\right)$	E $\left(lbf / in^2\right)$	AE (lbf)	Y (in)	AEY (lbf in)
Epoxy Fiberglass	0.50	$2.0\ (10^6)$	$1.0\ (10^6)$	0.80	$8.0\ (10^5)$
Aluminum	0.75	$10.0\ (10^6)$	$7.5\ (10^6)$	0.30	$2.25\ (10^6)$
Total			$8.5\ (10^6)$		$3.05\ (10^6)$

$$\overline{Y} = \frac{\sum AEY}{\sum AE} \tag{2}$$

$$\overline{Y} = \frac{3.05\,(10^6)\quad lbf\ \ in}{8.5\,(10^6)\qquad lbf} \tag{3}$$

$$\overline{Y} = 0.359\ \text{ inch} \quad \text{AE centroid} \tag{4}$$

Now let c equal the distance from the center of mass to the overall AE centroid.

For the epoxy section,

$$c_1 = \left|\overline{Y} - y_1\right| \tag{5}$$

$$c_1 = \left|\ 0.359 - 0.800\ \right| \tag{6}$$

$$c_1 = 0.441 \quad \text{inch} \tag{7}$$

For the aluminum section,

$$c_2 = \left|\overline{Y} - y_2\right| \tag{8}$$

$$c_2 = \left|\ 0.359 - 0.300\ \right| \tag{9}$$

$$c_2 = 0.059 \quad \text{inch} \tag{10}$$

The area moment of inertia for a rectangular section is

$$I = \frac{1}{12}[\text{ base }]\left[\text{height}^3\right] \tag{11}$$

For the epoxy section,

$$I_1 = \frac{1}{12}[\,1.25\,]\left[0.40^{\,3}\right] \quad \text{in}^4 \tag{12}$$

$$I_1 = 0.0067 \quad \text{in}^4 \tag{13}$$

For the aluminum section,

$$I_2 = \frac{1}{12}[\,1.25\,]\left[0.60^{\,3}\right] \quad \text{in}^4 \tag{14}$$

$$I_2 = 0.0225 \quad \text{in}^4 \tag{15}$$

The composite stiffness EI requires two terms, as calculated in Table 2 and in the following equations.

Table 3. Composite Stiffness

Section	c (in)	c^2 (in^2)	AE (lbf)	AEc^2 (lbf in^2)	E (lbf / in^2)	I_o (in^4)	$E_o\ I_o$ (lbf in^2)
Epoxy	0.441	0.194	$1.0\ (10^6)$	$1.94\ (10^5)$	$2\ (10^6)$	0.0067	$1.34\ (10^4)$
Alum.	0.059	0.0035	$7.5\ (10^6)$	$2.61\ (10^4)$	$10\ (10^6)$	0.0225	$2.25\ (10^5)$
Total				$2.20\ (10^5)$			$2.384\ (10^5)$

The composite beam stiffness factor EI is

$$EI = \sum AEc^2 + \sum E_o \ I_o \tag{16}$$

$$EI = 2.20\,(10^{5}) \ + \ 2.384\,(10^{5}) \qquad (\text{lbf in}^2) \tag{17}$$

$$EI = \ 4.58\,(10^{5}) \qquad (\text{lbf in}^2) \tag{18}$$

Perform a unit conversion.

$$EI = \ 4.58\,(10^{5}) \quad \left[\text{lbf in}^2\right]\left[\frac{\text{slugs ft}}{\text{lbf sec}^2}\right]\left[\frac{32.2\ \text{lbm}}{1\text{slug}}\right]\left[\frac{12\ \text{in}}{\text{ft}}\right] \tag{19}$$

$$EI = 1.77\,(10^{8}) \quad \left[\frac{\text{lbm in}^3}{\text{sec}^2}\right] \tag{20}$$

The mass per length for the epoxy fiberglass section is

$$\rho_1 = \left[0.065\ \frac{\text{lbm}}{\text{in}^3}\right]\left[1.25\ \text{in}\right]\left[0.40\ \text{in}\right] \tag{21}$$

$$\rho_1 = \left[0.033\ \frac{\text{lbm}}{\text{in}}\right] \tag{22}$$

The mass per length for the aluminum section is

$$\rho_2 = \left[0.100\ \frac{\text{lbm}}{\text{in}^3}\right]\left[1.25\ \text{in}\right]\left[0.60\ \text{in}\right] \tag{23}$$

$$\rho_2 = \left[0.075\ \frac{\text{lbm}}{\text{in}}\right] \tag{24}$$

The composite mass per length is

$$\rho = \rho_1 + \rho_2 \tag{25}$$

$$\rho = \left[0.033 \ \frac{\text{lbm}}{\text{in}} \right] + \left[0.075 \ \frac{\text{lbm}}{\text{in}} \right] \tag{26}$$

$$\rho = \left[0.108 \ \frac{\text{lbm}}{\text{in}} \right] \tag{27}$$

Furthermore, let the length be L = 20 inch.

$$f_n = \frac{\pi}{2L^2} \sqrt{\frac{EI}{\rho}} \tag{28}$$

$$f_n = \frac{\pi}{2\,[20 \text{ in}]^2} \sqrt{\frac{1.77\,(10^8) \left[\frac{\text{lbm in}^3}{\text{sec}^2} \right]}{0.108 \left[\frac{\text{lbm}}{\text{in}} \right]}} \tag{29}$$

$$f_n = 159.0 \ \text{Hz} \tag{30}$$

Reference

1. Dave Steinberg, Vibration Analysis for Electronic Equipment, Second Edition, Wiley, New York, 1988.