Column Buckling

Introduction

Structural dynamics problems deal with structures in motion. Examples include shock and vibration isolators, wind-induced deflections, aeroelastic phenomena, etc. One problem that does not involve motion, but which is closely related to structural dynamics, is column buckling. The similarity is due to the type of mathematics that is encountered in buckling analysis.

Buckling Defined

When a perfect column is subjected to a compressive axial force as shown in Figure 1, the only deformation that takes place is a shortening of the column. For low values of F, if the column were to be deflected laterally by a force perpendicular to the column, and the lateral force thereafter removed, the column would return to its straight position, even with the force F remaining in place. This indicates a condition of stability. If the load F were increased, there is a value of F for which, when the lateral load is removed, the column would remain in the deformed shape. This condition is referred to as buckling and the column is said to have failed from a structural standpoint. An example is given in Figure 1, where the column failure was due to an earthquake. Buckling can also be described in simple terms as bending or bowing of a column due to a compressive load. This is illustrated in Figure 2.

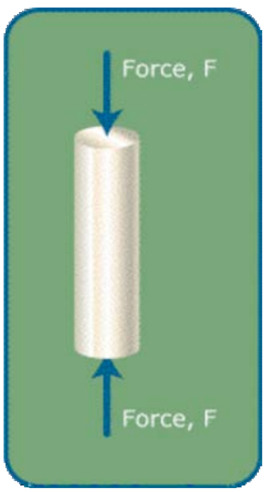

Figure 1

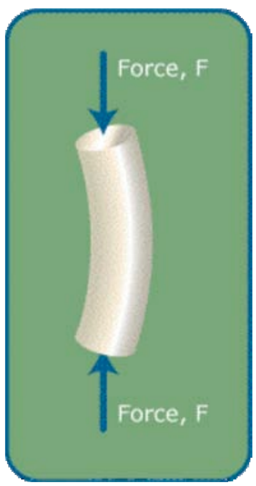

Figure 2

The buckling just described is termed primary instability because the phenomenon occurs without there being any distortion in the cross section. Secondary instability, another type of instability, can also occur. This can be demonstrated by buckling a soda straw, where the straw kinks at a point along its length. The failure is even more dramatic than the onset of the failure that is displayed by the primary instability. Both of these phenomena can be explored experimentally.

Euler's Formula

Euler analysis applies to slender columns only. The formula for the critical axial concentric load that causes the column to be on the point of collapse for one end fixed and the other end free is given by

$$P_{cr} = \frac{\pi^2 El}{4L^2},$$

where P is the load, E is the modulus elasticity of the material, I is moment of inertia, and L is the length of the material (column).

Experiment (Column Buckling)

Materials Needed
1. Soda straw
2. Wooden dowel or popsicle stick
3. Epoxy
4. Weight (coins, small bolts or washers)

Use the soda straw as the column. Epoxy one end of the soda straw to some type of platform (piece of wood, cardboard, etc). This will insure stability. Once glued, the soda straw should stand in the upright position. Epoxy the wooden dowel or popsicle stick to the free end of the soda straw. Diagrams of the experiment are given in Figure 3 and 4

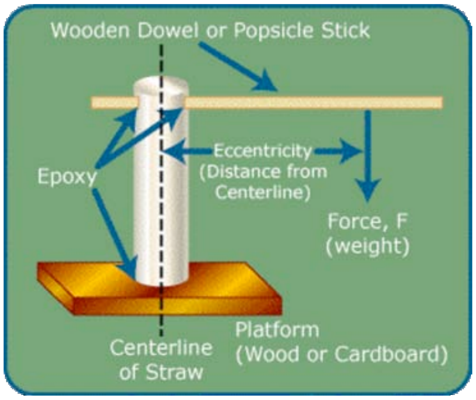

Figure 3

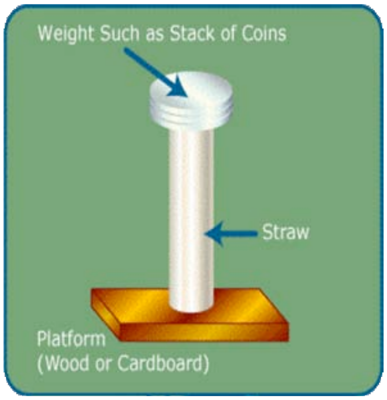

Figure 4

(a) Begin the experiment by placing the weights (such as coins) directly on top of the straw (Figure 4). See if you can get the straw to buckle.

(b) Next, take one weight and place it at different positions along the wooden dowel or popsicle stick (Figure 3). When a critical distance from the straw centerline is reached, the straw will buckle. Record the weight and the distance from the straw centerline. Repeat part (b). of the experiment for different weights, and the record the weight and distance from the straw centerline. You should observe that the heavier weights require smaller distances from the centerline for buckling to occur. Plot a graph of buckling load (weight applied) vs. distance from the straw centerline.

For the advanced student

In part (a) of the experiment (weight placed directly on top of the straw at the centerline), compare the buckling load (weight causing bending or bowing of the straw) to the Euler formula:

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

To do this, EI, which is related to the stiffness of the straw, must be measured as follows. Turn the straw and its platform (block of wood or cardboard) on its side and secure it to the edge of a table or desk, or to a table leg (Figure 6). For a leg support, c-clamps can be used to attach the platform.

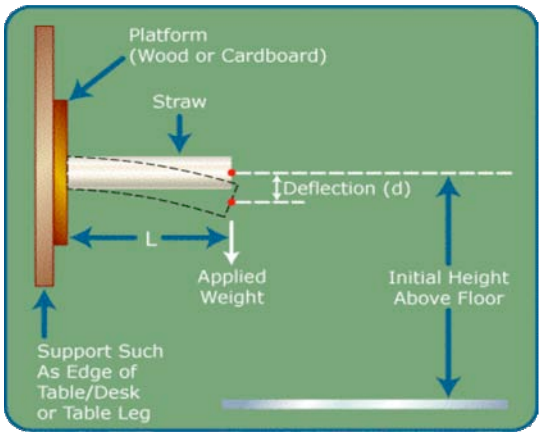

Figure 5

Use a yardstick or ruler to measure the height of the end of the straw above the floor. Hang a weight on the end of the straw and measure the amount of downward deflection of the straw end. (This deflection is the original height of the straw end above the floor minus the new height with weight applied.)

The bending stiffness **EI** is given by

$$EI = \frac{PL^3}{3d},$$

where **P** is the weight attached to the straw end, **L** is the length of the straw, and **d** is the deflection. The experiment can be repeated with different weights to get an average **EI.** Finally, calculate the buckling load using this average EI in the Euler formula

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

and compare to the buckling load that you obtained in part a of the experiment (weight applied to top of straw at centerline).

References

Column buckling is from an experiment based on material developed by Prof. Joseph A. Betz at the State University of New York at Farmingdale.